Minimization of FSA

Data Structures and Algorithms for Computational Linguistics III (ISCL-BA-07)

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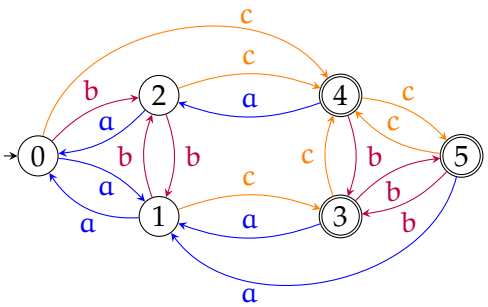
DFA minimization

- For any regular language, there is a unique *minimal* DFA
- By finding the minimal DFA, we can also prove equivalence (or not) of different FSA and the languages they recognize
- In general the idea is:
  - Throw away unreachable states (easy)
  - Merge equivalent states
- There are two well-known algorithms for minimization:
  - Hopcroft's algorithm: find and eliminate equivalent states by partitioning the set of states
  - Brzozowski's algorithm: 'double reversal'

Finding equivalent states

Intuition



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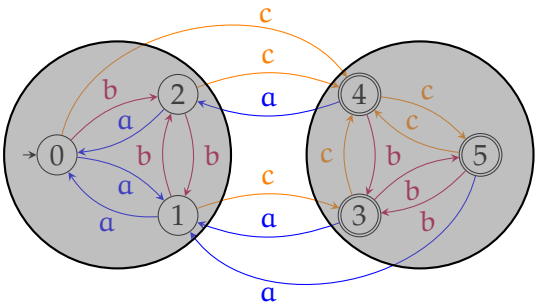

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Their *right languages* are the same.

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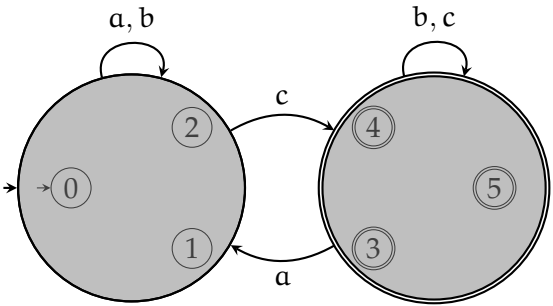

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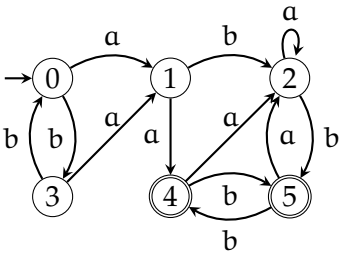
Minimization by partitioning



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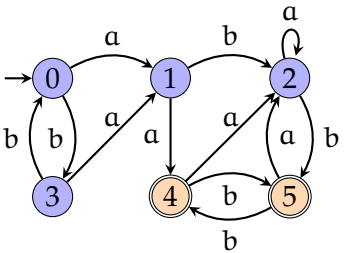

- Accepting & non-accepting states form a partition
  $Q_1 = \{0, 1, 2, 3\}$, $Q_2 = \{4, 5\}$

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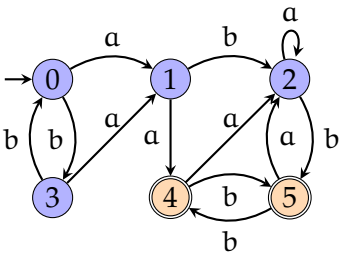

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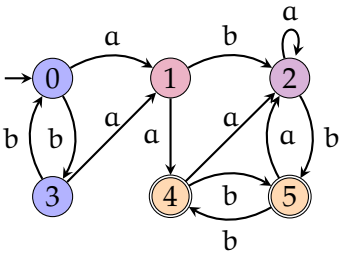

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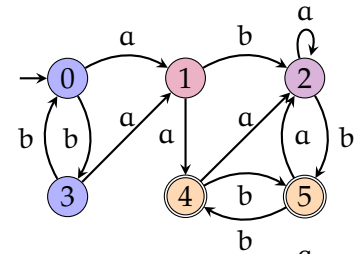

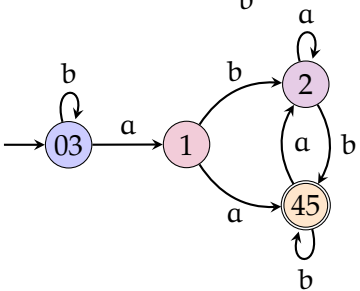

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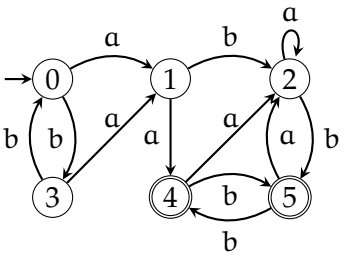
- If any two nodes go to different sets for any of the symbols split
- $Q_1 = \{0, 3\}$, $Q_3 = \{1\}$, $Q_4 = \{2\}$, $Q_2 = \{4, 5\}$
- Stop when we cannot split any of the sets, merge the indistinguishable states

Minimization by partitioning

tabular version



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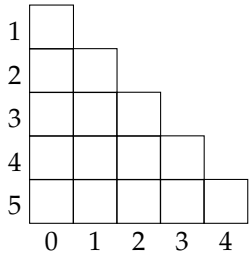

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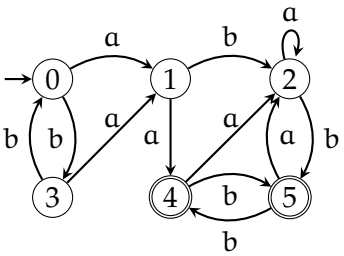

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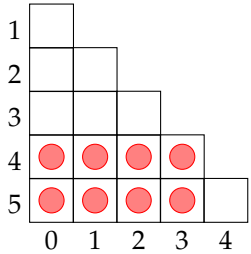

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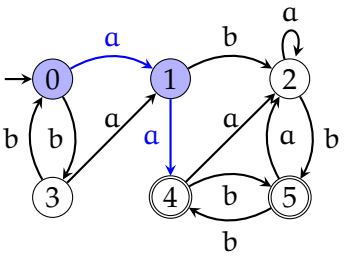

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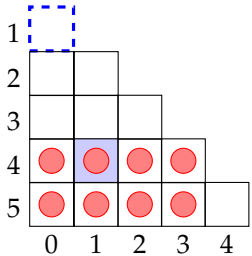

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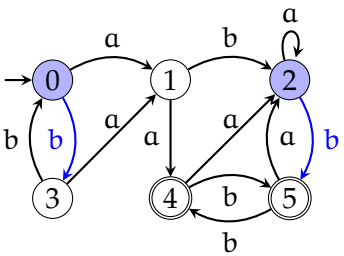

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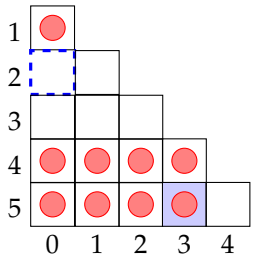

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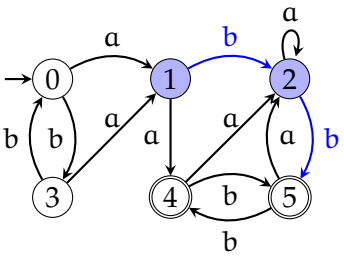

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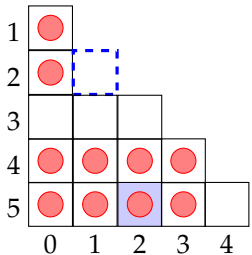

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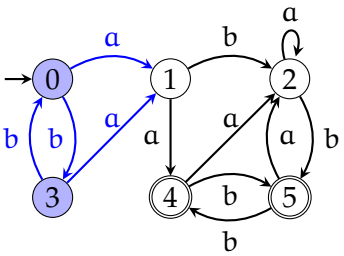

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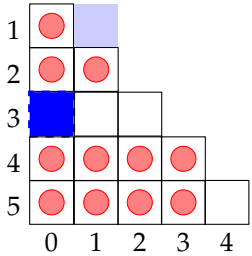

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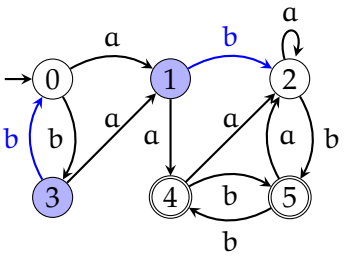

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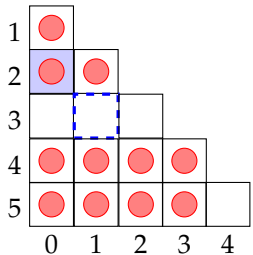

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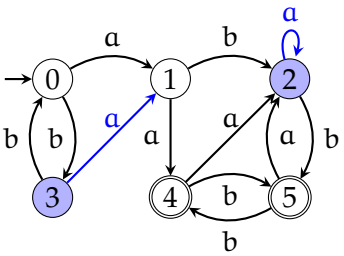

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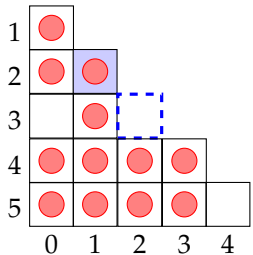

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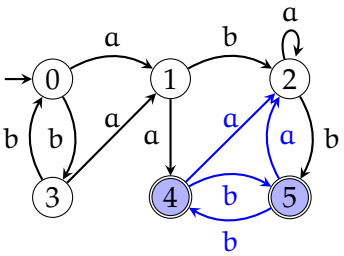

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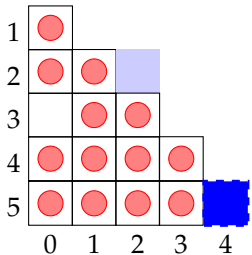

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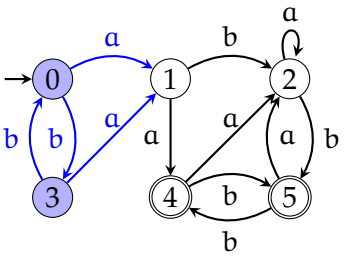

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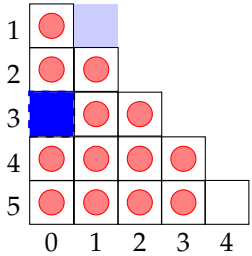

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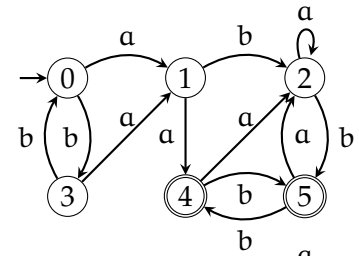

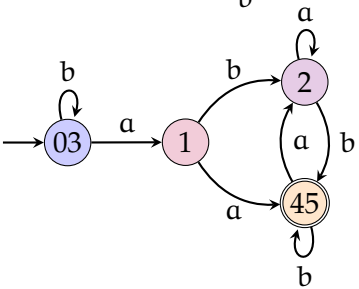

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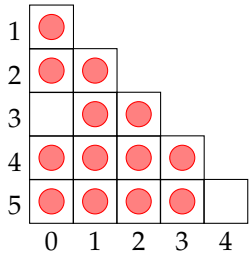

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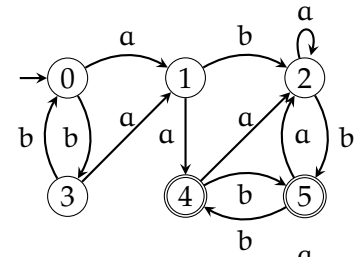

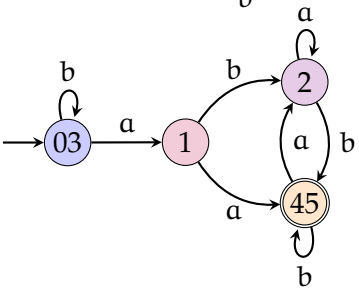

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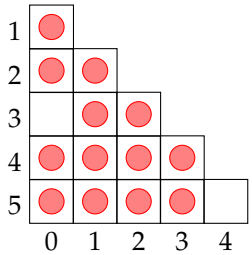
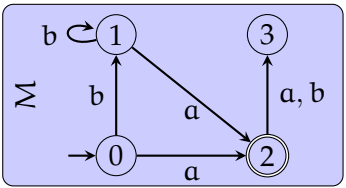


- Merge indistinguishable states
- The algorithm can be improved by choosing which cell to visit carefully

Brzozowski's algorithm

double reverse (r), determinize (d)



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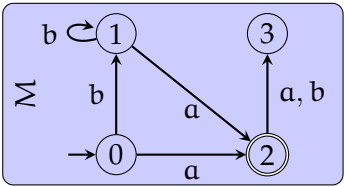

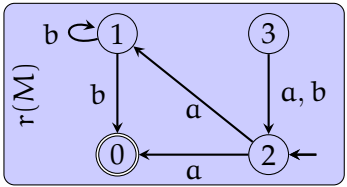

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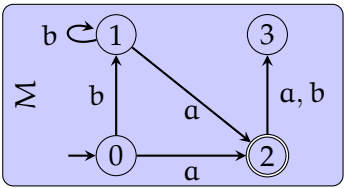

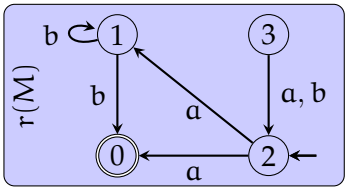

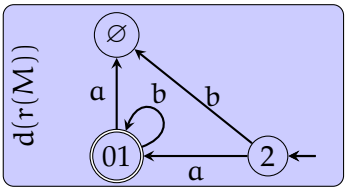

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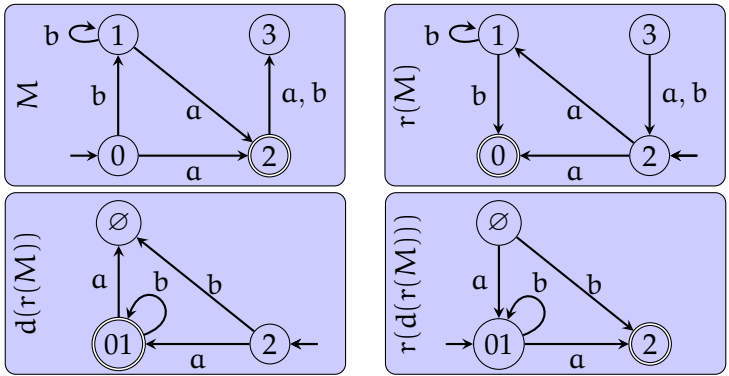

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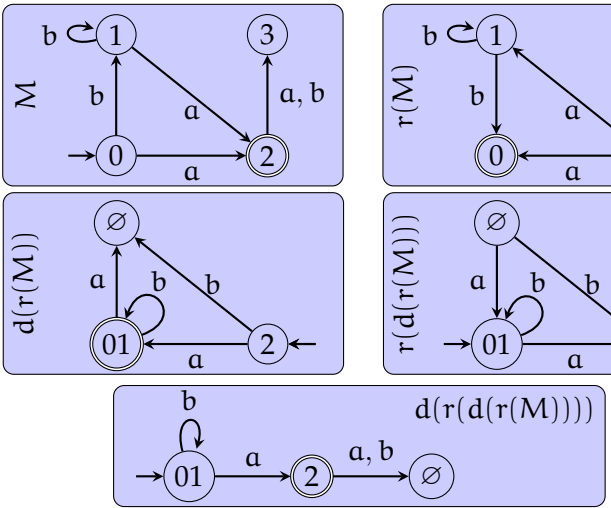
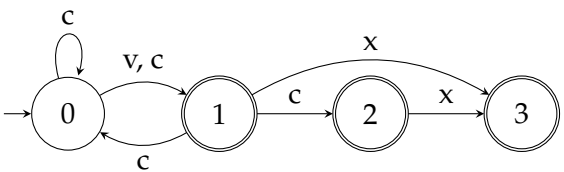


An exercise

find the minimum DFA for the automaton below



Minimization algorithms

final remarks

- There are many versions of the 'partitioning' algorithm. General idea is to form equivalence classes based on *right-language* of each state.
- Partitioning algorithm has $O(n \log n)$ complexity
- 'Double reversal' algorithm has exponential worst-time complexity
- Double reversal algorithm can also be used with NFAs (resulting in the minimal equivalent DFA – NFA minimization is intractable)
- In practice, there is no clear winner, different algorithms run faster on different input
- Reading suggestion: **hopcroft1979**, **jurafsky2009**

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Next:

- FST
- FSA and regular languages

Acknowledgments, credits, references